Young Children Talking in Mathematics: What is the Point of 'That'?

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As part of an intervention project to encourage exploratory talk with young children in mathematics, it was found that, although the children did not engage fully in reasoning, the intervention had supported some children in developing more cohesive discourse. The cohesion was evidenced through the children's use of deictic words, in particular the word 'that'. Examples of dialogue are contrasted to illustrate the changes in this use of deixis and are related to children's meaning-making in mathematics.

The focus of the paper is on how young lower attaining children developed cohesion and meaning-making within spontaneous pupil-pupil talk in mathematics. This is illustrated with dialogue that arose from an intervention project funded by the Esmee Fairbairn Foundation, and carried out in the southwest of England. The project involved ten classroom teachers in developing intervention strategies that would encourage exploratory talk in group work with young lower attaining children (aged 6 to 7 years old).

Exploratory talk was first noted by Barnes (1976), and has since been defined by Mercer, Wegerif, and Dawes (1999) as "a way of using language effectively for joint, explicit, collaborative reasoning" (p.97). Previous research has shown that explicit teaching strategies to encourage exploratory talk have supported the development of reasoning in verbal tasks (Rojas-Drummond, Perez,Velez, Gomez, & Mendoza, 2003) and in attainment in mathematics (Mercer & Sams, 2006). These earlier studies had been carried out with older pupils, so the intervention project referred to in this paper required the teachers to make adaptations of the strategies for exploratory talk with younger children.

Rojas-Drummond, Gomez, and Velez (2008) have further defined types of exploratory talk as elaborate and incipient. In elaborate exploratory talk counter viewpoints are given and arguments are reasoned and justified; in doing so pupils generalise and develop logical coherence. This would indicate the children are making meaning of the mathematics they are engaged in. In incipient exploratory talk arguments are rudimentary and relate to specific examples and tasks. Generalisation and logical coherence is not clearly evidenced. The use of the adjective incipient suggests the beginnings of exploratory talk. A question is raised whether these rudimentary arguments can still relate to meaning making.

In incipient exploratory talk rudimentary arguments are often supported in deixis. The term deixis comes from the Greek meaning to point via language (Rowland, 1992). Pronouns such as 'this' and 'that' are deictic as new meanings are created within use, as the words refer to something different in each situation. Where arguments are supported in deixis, then opinions are implicit within the context of discourse and are evidenced in actions such as pointing (Rojas-Drummond, 2008).

Use of deixis in children's rudimentary arguments was related to a functional linguistic perspective; how language was used to make meaning, or, how meaning-making emerged through the children's discourse. For pupil-pupil talk to be effective in supporting learning, there is a need for the children to make connections and to share ideas and meanings as they communicate (Gee, 1999). In this paper deixis was seen as a cohesive device in

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making connections and sharing meaning. It is proposed that meaning-making emerged through practice and discourse and that the use of deixis within incipient exploratory talk performed a role as a 'referential mechanism' (Radford, 2002).

Objectification and Deixis

A study of meaning-making that emerges through practice and discourse is consistent with Vygotskyan socio-cultural theories of mathematics education (Ernest, 1998; Lerman, 2001). Social meaning is seen as a precursor to conceptual individual meaning (Seeger, 2011) and is mediated through language. Social views of meaning emphasise the role of semiotics, both within and beyond verbal discourse. Radford's (2006) theory of knowledge objectification refers to the process of learning mathematics as socially and semiotically mediated, but takes account of non-verbal gestures as well as verbal communication; "ideas and mathematical objects ... are culturally embodied reflective, mediated activity" (p.42).

The definition of a mathematical object is used in a wide sense as "any entity which is involved in some way in mathematical practice or activity" (Font, Godino, & Gallardo, 2013, p.108). Hence this refers not only to abstract concepts but to a property, a representation, a procedure and so on. From Radford's (2002) perspective objectification is defined as "a process aimed at bringing something in front of someone's attention or view" (p.14), and that something includes the use of "objects, artifacts, linguistic devices, and signs that are intentionally used by individuals in social processes of meaning production" (p. 14). Radford (2003) referred to the use of pointing and deixis to "clarify intentions" and to "make apparent something new" (p.18), and hence to achieve objectification. Radford termed this *objectifying deixis* in that "it supports a powerful referential mechanism" (Radford, 2002, p. 18) by bringing mathematical objects to joint, shared attention.

From these theories of emergence, objectification is not just about communication with mathematical objects, but is about how mathematical objects are produced as referents through mathematical practices. Objectification happens as a mathematical object is brought to attention, and this is seen as happening within the discourse, both as use of language and through embodiment. Hence, in this study, the use of deixis as a pointing reference was seen, not just as connection building within a group, but also in producing referents of mathematical objects that were shared between the children.

Rowland's (1992; 1999; 2000) research into the use of language in mathematics has considered children's deictic use of pronouns. Rowland examined how 'it' was used as a generality. Radford's (2002) studies of deixis in objectification have recognised the deictic use of words 'this' and 'that' as key elements in "mathematical discursive meaning production processes" (p.15) where their "primary function is to point to something in the visual field of the speakers" (p.17). In this paper, the children's uses of the pronoun 'it' and the demonstrative 'that' were examined by comparing the pupil-pupil talk from pre-intervention and post-intervention sessions.

The Study

Ten teachers participated in the project over one school term. Each of the ten teachers was asked to select six focus children and to engage these children in independent group work at least twice a week. This was managed in two smaller groups of three children (or triads). Lessons were video-taped in each class over the term. These video tapes included a focus on one of the triads as they worked on a task in each of the lessons.

The approach of the intervention was from a practical perspective (Carr and Kemmis, 1993), and was based on the use of explicit strategies known to support exploratory talk. The researchers worked with the teachers in using these strategies in a non-prescribed way, so that they could be adapted by the teachers to support the children in their classes. A key strategy in developing exploratory talk was that the children agreed on a solution and gave reasons why they did or did not agree. Along with the strategies to support talk, the teachers developed and used mathematical tasks as they saw appropriate within the context of their classroom. These tasks were intended to create a context where the children would agree on solutions.

Initially analysis was carried out with the ten classrooms through examination of the independent pupil-pupil talk from the videos of the pre-intervention session and one post-intervention session. The amount of independent pupil-pupil talk in the pre-intervention sessions varied from almost none at all in four classrooms to some pre-intervention independent talk in two classrooms, and there were four classrooms where this was already an established norm. In the four classrooms where there had been no pupil-pupil talk in the pre-intervention sessions, it was not possible to do a comparison, so six groups were used in comparison analysis.

Results

The children's use of deictic words was examined across the six comparison triads as they engaged in independent pupil-pupil talk about mathematics. As part of the analysis a word frequency query was carried out using NVivo 9 software in determining the twenty highest frequency words. This was carried out with word stops for content words that would suggest a context or a name. The aim was to examine function words that suggested meaning in use generally, rather than meaning within a specific context or mathematics topic. The function words 'a' and 'and' were also stopped as their uses were not seen as important, and they are not deictic.

The word frequency query suggested that the pronouns 'that' and 'it' were used frequently by the children in their mathematics talk, both before and after the intervention. However the word 'that', in particular, became the most frequent function word following the intervention. The word 'it' increased in frequency slightly following the intervention. The pronoun 'you', the conjunction 'so', and the verb 'do' also featured as high frequency words, but there is not enough space to examine these words, also 'so' and 'do' are not deictic. This short paper focuses on the use of the function words 'that' and 'it' as they were used in deixis. Examples of dialogue from pre-intervention and post-intervention sessions are given to illustrate changes in the children's use of these words.

Group 1: Diane, Emma and Olwen

In the pre-intervention group session the task was to model or show visually a function such as 'double seven' or 'half of sixteen'. The children were given a large card with an image of two baskets and a set of images of eggs that they could manipulate and stick onto

the baskets. They had a set of cards with the functions. The task was to model the function by sticking the eggs onto the images of the baskets.

In the first extract (Figure 1) the children had a card with the function 'double seven'. The children decided to stick seven eggs in each of the two baskets. Olwen stuck the eggs onto the basket picture and they realised they needed to give the total number of eggs.

5. Diane: No, we need to know what ***it*** equals (Diane points to the two baskets)
6. Olwen: Seven times...Oh yea, Emma's good at ***that*** (Olwen looks at Emma)
7. Olwen: Seven fourteen two, fourteen ....equals (Olwen seems to be using knowledge of doubling)
8. Diane: Fourteen (Diane is pointing to the eggs to count them and gives an answer)
9. Olwen: Yes, exactly, you don't have to count ***it***, just work ***it*** out.

Figure 1: Group 1 pre-intervention session

The children were working together to show a model of the function but they worked individually to find solutions to the total number of eggs. Although there was a suggestion that Emma could give the answer (maybe Olwen perceived that Emma knew some multiplication facts) Emma did not volunteer an answer. Diane gave the answer using a counting strategy and Olwen appeared to be using knowledge of facts. Olwen then explained that there was no need to use a counting strategy but she did not explain how she 'worked ***it*** out'.

The use of 'that' in the phrase 'Emma's good at ***that***' referred to a process, in this case double the number of eggs in one basket. The use was anaphoric in that it referred to the problem posed previously. The use was not spatial and there was no gesture towards an object. This is compared with the children's use of 'it'. In utterance 1 Diane was pointing to the two baskets and we can assume the referent was the total number of eggs. 'It' did not relate to the eggs as objects but to the need to find the total. This use was repeated in utterance 5 when Olwen referred to 'work ***it*** out' to find the total.

In the post-intervention session (Figure 2), the three children were given a set of cards with word problems written, and a set of cards with representations of number lines that showed different calculations. The task was to match a word problem with a number line representation. Following the intervention strategies for exploratory talk the children were encouraged to agree on each of the matchings.

In Figure 2, Olwen read out the first word problem. Emma pointed to two different number line representations asking Diane which one she thought was the correct match. Emma then made a decision herself and Olwen agreed. Emma checked that Diane agreed. Emma then placed a different word problem and a number line representation in front of Diane. This second word problem was not read out but Diane referred to a number line representation and skip counted on the number line to check. Olwen joined in the skip counting.

In Figure 2 the children were working together to agree which representations could be paired together. This required the children to point to the different representations in order to indicate to the others which one they meant. In utterances 1-5 of the dialogue 'that' was used frequently, alongside the gesture of pointing, in referencing to the number line representations. In utterances 6 and 7, Diane and Olwen skip counted on the number line.

Diane started, and Olwen completed the skip count, suggesting that they were thinking closely together. Although there was still no explanation of why 'it's ***that*** one', the increased use of spatial deixis suggested that there was cohesion in the dialogue. The children used 'it' in saying '***it***'s that one' or '***it***'s this one'. We can assume that 'it' is referring to the correct representation.

1. Emma: 15 flies were on a cake, 5 more came along, how many were there altogether? (Emma reads out the word problem on the card). Do you think ***it***'s ***that*** one or ***that*** one Diane? (Emma points to two different number line representations). ***That*** one (Emma points to one of the number line representations).
2. Olwen: ***It***'s definitely ***that*** one Emma (Olwen points to the same representation)
3. Emma: Do you think ***it***'s ***that*** one? (Emma checks with Diane)
4. Diane: ***It***'s ***that*** one.
5. Emma: So do you think ***that*** one might go with ***that*** one? (Emma places another pair of word problem and number line representation in front of Diane. In this case the word problem is 'A ladybird has 6 legs, how many legs would four ladybirds have?').
6. Diane: I don't know if ***it***'s ***this*** one, I'm checking, 6, 12... (Diane is looking at this further representation for the new word problem and skip counting on the number line).
7. Olwen: 18... 24 (Olwen continues skip counting on the number line)

Figure 2: Group 1 post-intervention session

Group 2: Fran, Iris and Pierce

In the pre-intervention session the children worked in pairs. One child picked a card with a multiplication problem and gave the answer. The other child then asked for an explanation. The teacher had modelled how to do this and started phrases with "Show me..." and "Why...?" It was Pierce's turn to give the explanation and he demonstrated skip counting on his fingers to show how he calculated 10 x 10. Pierce also referred to the multiplication sign as meaning 'times', and it would seem that in the phrase 'So if ***it*** says first then ***it*** turns out whatever ***it*** is' he was referring to the multiplicand and the multiplier.

1. Ben: Ten, ten tens what's ***that***? (Ben has picked a card with 10 x 10)
2. Pierce: You can count up in your fingers, so imagine 10 20 30 40 50 60 70 80 90 100 like ***that*** (Pierce counts ten as a unit for each finger ten times) and if ***it***'s on there, ***that*** means times (Pierce points to the multiplication sign on the card). So ten times. So if ***it*** says first then ***it*** turns out whatever ***it*** is. So ***it*** will be a hundred. (Ben nods his head).
3. Ben: So can you do ***it*** to me now? You pick up one. (Pierce picks out a card for Ben)

Figure 3: Group 2 pre-intervention session

In Figure 3, the pre-intervention session for group 2, ‘that’ was used three times. In utterance 1 ‘ten tens what’s ***that***?’ it was not clear if Ben was referring to the answer or the explanation. However as the teacher had modelled how to give an explanation Pierce may have interpreted this as the explanation. The use was anaphoric and not spatial deixis. In utterance 2 ‘so imagine 10 ... 100 like ***that***,’ Pierce was also using the word in an anaphoric sense to refer to what he had just said. In both cases there was no accompaniment with the gesture of pointing. In utterance 2 Pierce continued ‘if ***it***’s on there, ***that*** means times.’ We can assume that ‘it’ and ‘that’ were both used as pronouns to refer to the multiplication sign, as Pierce pointed to the multiplication sign. Both ‘it’ and ‘that’ were used in a spatial deictic sense. There was further use of ‘it’ in a deictic sense in Pierce’s reference to the multiplicand and multiplier. In utterance 3 Ben then used ‘it’ as a pronoun to refer to the task of giving an explanation.

In the post-intervention session the children were given a sheet of paper with inequalities >50 on one side and <50 on the other. They were told to write numbers on each side that would be true for each inequality. They talked together to decide what the > sign meant. They referred to other recordings of the signs of > and < that were recorded on a white board when they had talked with their teacher earlier.

1. Fran: ***That***’s the sign (Fran points to the > sign on the sheet)
2. Iris: Oh yeah
3. Iris: Is ***that*** more than or less than, which one? (Iris points to the > sign)
4. Pierce: ***That*** is. (Pierce points to the > sign)
5. Fran: If we look at, if we have a look, ***that***’s less than... (Fran picks up the whiteboard with the recordings and points to the < sign)
6. Iris and Fran: ***That***’s more, ***that***’s more (Iris and Pierce are pointing to the > sign)
7. Pierce: ***That***’s more, more than (Pierce points to the >sign)

Figure 4: Group 2 post-intervention session

In Figure 4, the Group 2 post-intervention session there was frequent use of the word ‘that’, as the children pointed to the inequality signs in deciding whether they meant ‘less than’ or ‘more than’. There was little evidence of extending ideas or explanations. At no point did they describe the signs or give an example of numbers they could use, although later in the task they did write numbers on each side of the sheet. Deixis was used for joint attention and the children came to an agreement on what the signs meant. In this dialogue the word ‘it’ was not used at all. The children were referring directly and specifically to the signs on the paper, and on the whiteboard.

Analysis and Discussion

It would seem that in the post-intervention dialogues the use of the word ‘that’ tended to take the place of the word ‘it’ as a deictic term. Although the words ‘it’ and ‘that’ can be interchangeable, ‘it’ is used in a less specific way. ‘That’ is used with more emphasis in pointing directly to a specific sign or representation that is physically present or “in the visual field of the speakers” (Radford, 2002, p.17).

Rowland (2000) had proposed that the pronoun ‘it’ is used as a generality and he gave examples of children’s use of ‘it’ as a pointer to a concept, an abstract idea that children can use to hold a generality, and to draw attention to a mathematical object that they

cannot name (Rowland, 1992). In the pre-intervention dialogues the use of 'it' could be seen to hold an idea. In Figure 1 the referent for 'it' was the total or the process in finding the total. In Figure 3 Pierce used 'it' as a generality in referring to whatever the multiplicand and the multiplier were. Pierce was using 'it' as a conceptual variable (Rowland, 2000) to hold the concept of the operation of multiplication, whatever the numbers were, without having to use the words. What was less clear from the pre-intervention Dialogues 1 and 3, was how much sense the other children was making as Olwen and Pierce referred to 'it'. In Figure 1 it would seem that the children arrived at the same answer, but Olwen's direction in utterance 5, that you 'just work ***it*** out', was not said in a way that the other children could make sense of. Pierce gave his explanation in Figure 3 but, although Ben was seen to nod his head, he gave no response to Pierce's explanation, he asked for the next turn.

In both the post-intervention tasks the children were engaged in choosing between different representations or signs. As such they needed to determine the meaning of the objects that were in the sight of themselves, as the speakers. A key specific strategy when introducing exploratory talk was the encouragement to agree. In these two examples the children were agreeing on the meanings of the signs or representations, and they were pointing directly to them. The children were focusing attention on physically present mathematical objects. This was different to the use of 'it' in deixis to hold a concept or a conceptual variable that was not physically present in the sight of the speakers.

This use of deixis in the post-intervention dialogues is consistent with Rojas-Drummond at al.'s (2008) notion of incipient exploratory talk. Little reasoning or justification was happening, in fact more reasoning seemed to happen in the pre-intervention dialogues, particularly with Pierce in Figure 3. As Rojas-Drummond et al. had characterised, the argument was rudimentary and opinions were evidenced in pointing. However in pointing to the different signs or representations the children were clarifying their understanding by pointing to an example. They were bringing the object in front of someone's attention, (Radford, 2002; 2003) and, possibly, clarifying their understanding together. The children may have found it difficult to explain mathematical objects verbally but they could point to an example. As with an ostensive definition, meaning was gathered by pointing to the example.

Conclusion

The use of deixis by these young children suggested that they could make meaning semiotically and within a specific situation. As Gee (1999) identified, "A situated meaning is an image or pattern that we assemble 'on the spot' as we communicate in a given context" (p.47). Whilst elaborate exploratory talk is seen as talk where children give justifications and reasons, incipient exploratory talk is characterised by giving opinions that are not explained or reasoned or justified. Within the literature on exploratory talk (Rojas-Drummond et al., 2003; 2008) incipient exploratory talk is seen as a stage towards elaborate exploratory talk.

From this study it was not possible to say if the children's use of incipient exploratory talk was a step towards elaborate exploratory talk, or if this type of talk was typical of their age or their lower attainment in mathematics. However this study has suggested that, even though there was a lack of argument or reasoning that would have suggested logical coherence, incipient exploratory talk could be seen as supporting meaning-making. From a socio-cultural perspective, a deixis-based type of talk could be related to situated, 'on the

spot' meaning, and would seem worthy of further study in children's developing discourse in mathematics.

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